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STEADY FLOW OF MICRO POLAR FLUID IN A PIPE WITH POROUS WALLS

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ABSTRACT

Livesy investigated the radial flow of incompressible fluid between the plates. The assumption of fixed velocity profile leads to an inconsistency which can be of significant consequence. Bird pointed out in the flow regime where the inertia forces are important. The velocity must depart from a parabolic pattern.this is true even at large radii where the effects are negligible. Savage solved the problem of source flow between parallel disks.

KEYWORDS: porous medium, rotating channel convective heat and mass transfer.

INTRODUCTION

The combined rotational and buoyancy effects are very common in nature. Most flows has regions of rotations as well as stratification. Buoyancy and rotational effects are often comparable in geophysical process. Convective transport in a rotating atmosphere over a heated surface gives rise to typhoons and other rising atmosphere circulations. The unsteady flow of a rotating viscous fluid has been studied by several authors to analyse the growth and development of boundary layer associated with geothermal flows for applications in geophysical fluid dynamics. [1-7] Rao have made an initial value investigation of the combined free and forced convection effects in an unsteady hydromagnetic viscous incompressible rotating fluid between two discs under a uniform transfers magnetic field. Nagaraja [5] has investigated combined effects of heat and mass transfer flow of a viscous incompressible fluid through a porous medium in a rotating horizontal channel bounded by the flat walls. Prasad [7] has studied the mixed convective heat and mass transfer flow of a viscous fluid through a porous medium in a rotating horizontal channel bounded by the process medium in a rotating parallel channel in the presence of a constant heat source.

In this paper we deal with the oscillatory flow of a combined effect of heat and mass transfer flow of a viscous incompressible fluid through a porous medium in a rotating horizontal channel bounded by flat walls. The perturbatioon in the flow is created by the nontensional oscillitons of the lower plate. The solutions of velocity field, temperature and concentration distributions are obtained. The shear stress, the rate of heat and mass transfer has been evaluated for different variations of the governing parameters.

FORMULATION OF THE PROBLEM

We consider the unsteady flow of an incompressible viscous fluid through a porous medium bounded by two parallel plates. In the undisturbed states both the plates and the fluid rotate with the same angular velocity (Ω) and are maintained at constant temperature and concentration. The lower plate performs nontensional oscillations in its own plane.

The plates are cooled or heated by constant temperature gradient in some direction parallel to the plane of the plates. We choose a cartesian coordinate system O(x,y,z) such that the plates are at z=0 and z=1 and the z axis coinciding with the axis of rotation of the plates. Neglect the soret and doffer effect, the unsteady hydrodynamic boundary layer equations of motions with respect to a rotating frame moving with angular velocity Ω are the momentum equations

$$\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho} \left(\frac{\partial p}{\partial x} \right) + v \left(\frac{\partial u^2}{\partial z^2} \right) - \left(\frac{v}{k} \right) u \tag{1}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho} \left(\frac{\partial p}{\partial y} \right) + v \left(\frac{\partial v^2}{\partial z^2} \right) - \left(\frac{v}{k} \right) v \tag{2}$$

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(3)

$$\mathbf{0} = -\frac{1}{\rho} \left(\frac{\partial p}{\partial z} \right) - g \left(1 - \beta (T - T_0) - \beta (C - C_0) \right)$$

The energy equation

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)(T - T_0) = \lambda \frac{\partial^2}{\partial z^2}(T - T_0) + v\left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{v}{k_1}\right)(u^2 + v^2) \right)$$
(4)

The diffusion equation

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)(C - C_0) = D_1 \frac{\partial}{\partial z^2}(C - C_0)$$
⁽⁵⁾

Where u, v are velocity components along x and y directions respectively, p is the pressure including the centrifugal force , ρ is the density, k is the permeability constant, μ is the coefficient of viscosity, λ is the thermal diffusivity, D₁ is the chemical molecular diffusivity, β is the coefficient of thermal expansion and β^* is the volumetric coefficient of expansion with mass fraction. Combining the equation (2.1) and (2.2) we obtain

$$\frac{\partial q}{\partial t} - 2i\Omega q = -\frac{1}{\rho} \left(\frac{\partial p}{\partial x} + i \frac{\partial p}{\partial y} \right) + v \frac{\partial q^2}{\partial z^2} - \left(\frac{v}{k} \right) q$$
(6)
Where q=u+iv
Integrating equation (2.3) we obtain

$$\frac{p}{\rho} = -gz + \beta g \int (T - T_0) dz + \beta g \int (C - C_0) dz + \Phi \left(\xi, \xi\right)$$
(7)
Where

$$\xi = x - iy, \bar{\xi} = x + iy$$
Using (7), equation (6) can be written as

$$\frac{\partial q}{\partial t} - 2i\Omega q - v \frac{\partial q^2}{\partial z^2} + \left(\frac{v}{k} \right) q = -2\beta g \frac{\partial}{\partial \xi} (T - T_0) - 2\beta g \frac{\partial}{\partial \xi} (C - C_0)$$
(8)
Since q = q (z,t), equation (2.8) is valid if the temperature and concentration distributions are of the form

$$T - T_0 = \alpha_1 x + \beta_1 + \theta_1(z, t)$$
(7)

Where $\alpha_1, \beta_1, \alpha_2, \beta_2$ are the gradients of the temperature and concentration along O(x,y) directions respectively, $\theta_1(z, t)$, $C_1(z, t)$ are the arbitrary functions of z and t. we take

$$T_{0} + \alpha_{1}x + \beta_{1}y + \theta_{1}w_{1} \text{ and } T_{0} + \alpha_{1}x + \beta_{1}y + \theta_{1}w_{2}, C_{0} + \alpha_{2}x + \beta_{2}y + C_{1}w_{1}$$
And $C_{0} + \alpha_{2}x + \beta_{2}y + C_{1}w_{2}$
To be temperature and concentration of lower and upper plates respectively, for t > 0.
Substituting (2.7) and (2.6) using (2.8) we get
$$\frac{\partial q}{\partial t} = 2i\Omega q + \frac{v}{k}q - \mu \frac{\partial q^{2}}{\partial z^{2}} + \beta g \overline{A}z + \beta g \overline{B}z = D_{2} \qquad (9)$$
Where $D_{2} = [\emptyset(\tau_{Y}, \overline{\tau}_{Y})]\tau_{Y}$
 $A = \alpha_{1} + i\beta_{1}$ and $B = \alpha_{2} + i\beta_{2}$
Introducing non dimensional variables (z, t, q, θ, c)
 $\dot{z} = \frac{z}{L}, \ \dot{t} = \frac{tv}{L^{2}}, \ \dot{q} = \frac{tv}{L^{2}}, \ \dot{w} = \frac{wL^{2}}{v^{2}}$
 $\dot{\theta} = \frac{\beta g L^{3}(\theta_{1} - \theta_{1}w_{1})}{v^{2}}$
The governing equations in the non dimensional form are
 $q_{zz} - (D^{-1} - 2iE^{-1})q - q_{1} = G(1 + N)z - R \qquad (10)$
 $P(\theta_{1} + G_{1}u + G_{2}v) = \theta_{zz} + E_{c}D^{-1}q.\overline{q} \qquad (11)$
 $S_{c}(C_{1} + G_{1_{c}}u + G_{2_{c}}\mu) = C_{zz} \qquad (12)$
Where
 $E = \frac{v}{L^{2}g}$
[Ekmann number]
 $D^{-1} = \frac{L^{2}}{k}$ [Darcy parameter]

$$(\boldsymbol{G}_1, \boldsymbol{G}_2) = \frac{\beta G L^4}{\gamma^2} (\boldsymbol{\alpha}_1, \boldsymbol{\beta}_1) [\text{Grashofnumber}]$$

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$$(G_{1c}, G_{2c}) = \frac{\dot{\beta}GL^4}{v^2} (a_2, \beta_2) [\text{modified Grashof number}] \\ R = \frac{k^3D}{v^2} [Reynolds number] \\ P = \frac{k}{\lambda} [P randtl number] \\ P = \frac{k}{\lambda} [P randtl number] \\ D^{-1} = \frac{k^2}{k_1} [Darcy number] \\ E_c = \beta gLP [Eckert number] \\ S_c = \frac{v}{D} [Schnidtparameter] \\ G = G_1 + (G_2, \infty) \\ G = G_1 + (G_2, G_2, 0) \\ G = G_1 + (G_2, G_1, 0) \\ G = G = G_1 + (G_2, G_1, 0) \\ G = G = G_1 + (G_2, G_1, 0) \\ G = G = G_1 + (G_2, G_1, 0) \\ G = G = G_1 + (G_2, 0) \\ G = G_1 + (G_2, 0) \\ G = G_1 + (G_2, 0) \\ G = G_2 = G_1 + (G_2, 0) \\ G = G_1$$

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The corresponding boundary conditions are	
$f_3 = 0$; $g_3 = 0$; $H_3 = 0$ on $z = 0$	
$f_3 = 0$; $g_3 = 0$; $H_3 = 0$ on $z = 1$	(30)
And	
$f_{4,zz} - (h^2 - 2iw)f_4 = 0$	(31)
$g_{4,zz} + (2iPw)g_4 = -E_c\left((f_2\bar{f}_2) - D^{-1}(f_2\bar{f}_2)\right)$	(32)
$H_{4,zz} + (2iS_cw)H_4 = S_cG_{1c1}Re(f_4) + G_{2c2}I_m(f_4)$) (33)
The corresponding boundary conditions are	
$f_4 = 0$; $g_4 = 0$; $H_4 = 0$ on $z = 0$	
$f_4 = 0; g_4 = 0; H_4 = 0 \text{ on } z = 1$	(34)
Solving the equations (15)-(17), (19)-(21), (23)-(25), and(26)-(28) with respect to the boundary conditions
(18),(22),(26) and (29), the solutions are	
$f = (A_1 + iA_2)(z^2 - z)$	
$f_1 = a(1-z) + B_1(z^2 - z) + iB_2(z^2 - z)$	
$f_2 = b(1-z) + D_1(z^2 - z) + D_2i(z^2 - z)$	
$f_3 = 0$	
$f_4 = 0$	
$g = c_5(z^2 - z) + z$	
$g_1 = (E_1 + iE_2)(z^2 - z)$	
$g_2 = c_7(z^2 - z)$	
$g_3 = c_8(z^2 - z)$	
$g_4 = c_9(z^2 - z)$	
$H = c_{10}(z^2 - z) + z$	
$H_1 = (L_1 + iL_2)(z^2 - z)$	
$H_2 = (M_1 + iM_2)(z^2 - z)$	
$H_3 = 0$	
$H_4 = 0$	

SHEAR STRESS, NUSSELT NUMBER AND SHERWOOD NUMBER

The non-dimensional shear stress τ_x and τ_y at the lower and upper plates are given by

 $\begin{aligned} (\boldsymbol{\tau}_{x} + \boldsymbol{\tau}_{y})_{z=0} = (\frac{\partial q}{\partial z})_{z=0} \\ (\boldsymbol{\tau}_{x} + \boldsymbol{\tau}_{y})_{z=0} = (\frac{\partial q}{\partial z})_{z=1} \end{aligned}$

 $(\mathbf{t}_{x} + \mathbf{t}_{y})_{z=0} - (\frac{\partial z}{\partial z})_{z=1}$ The rate of heat transfer coefficient (nusselt number) on the plates is given by $(\mathbf{Nu})_{z=0} = (\frac{\partial \theta}{\partial z})_{z=0} = (\frac{\partial g}{\partial z})_{z=0} + (\frac{\partial g_{1}}{\partial z})_{z=0} \mathbf{e}^{iwt} + (\frac{\partial g_{2}}{\partial z})_{z=0} \mathbf{e}^{-iwt} + (\frac{\partial g_{3}}{\partial z})_{z=0} \mathbf{e}^{2iwt} + (\frac{\partial g_{4}}{\partial z})_{z=0} \mathbf{e}^{-iwt}$ $(\mathbf{Nu})_{z=1} = (\frac{\partial g}{\partial z})_{z=1} = (\frac{\partial g}{\partial z})_{z=1} + (\frac{\partial g_{1}}{\partial z})_{z=1} \mathbf{e}^{iwt} + (\frac{\partial g_{2}}{\partial z})_{z=1} \mathbf{e}^{-iwt} + (\frac{\partial g_{3}}{\partial z})_{z=1} \mathbf{e}^{2iwt} + (\frac{\partial g_{4}}{\partial z})_{z=1} \mathbf{e}^{-iwt}$ The rate of mass transfer (sherwood number)on the plates are given by $(\mathbf{sh})_{z=0} = (\frac{\partial C}{\partial z})_{z=0} = (\frac{\partial H}{\partial z})_{z=0} + (\frac{\partial H_{1}}{\partial z})_{z=0} \mathbf{e}^{iwt} + (\frac{\partial H_{2}}{\partial z})_{z=0} \mathbf{e}^{-iwt} + (\frac{\partial H_{3}}{\partial z})_{z=10} \mathbf{e}^{2iwt} + (\frac{\partial H_{4}}{\partial z})_{z=0} \mathbf{e}^{-iwt}$ $(\mathbf{Sh})_{z=1} = (\frac{\partial C}{\partial z})_{z=1} = (\frac{\partial H}{\partial z})_{z=1} + (\frac{\partial H_{1}}{\partial z})_{z=1} \mathbf{e}^{iwt} + (\frac{\partial H_{2}}{\partial z})_{z=1} \mathbf{e}^{-iwt} + (\frac{\partial H_{3}}{\partial z})_{z=1} \mathbf{e}^{2iwt} + (\frac{\partial H_{4}}{\partial z})_{z=1} \mathbf{e}^{-iwt}$

DISCUSSION ON NUMERICAL RESULTS

The oscillatory solution for the velocity, temperature and concentration have been computed numerically for the governing parameters D-1,N, w and their profiles are drawn in figs1-13.for computational purpose we have assumed G to be real so that the applied pressure gradient in the y-direction is zero. Also the Prandtl number P is taken to be 0.71.Since thermal buoyancy balances the vertical pressure gradient in the absences of any other applied forces in the direction of rotation, the flow takes place in planes parallel to the boundary plates. The flow is three dimensional and all the perturbed variables have been obtained using boundary layer equations which would reduce to three coupled partial differential equations for a complex velocity ,temperature and concentration.



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Figs 1-7 corresponds to profiles of the axial velocity when one of the plates (lower) oscillates with given amplitude and the other at rest. The imposed pressure gradient along x-direction is choosen to be negative. The actual flow along x-axis remains negative for values of G.we find that in all cases ,U rises from its prescribed value on the lower plate to the maximum attained in the lower half and gradually reduces to rest on the upper plate. The magnitude of U experiences a depreciation in N and w (figs.1-2).when the concentration buoyancy dominates over the thermal force U experiences an enhancement or a reduction according as the two forces either act in the same or in opposite directions(fig-2).In contrast to U, the transeverse velocity V is positive for different N&w.we find that V dicreases with w.The resultent velocity profiles like its components are bell shaped curves with their maximum attained at y=-150 in the lower region.The magnitude of the resultant velocity dicreases with w (fig6).Also we observe that the resultant velocity increases with N(>0).

The temperature profiles are plotted in figs.(7-9).We find that the temperature gradually dicreases from its value on the lower plate to attain its minimum and then again increases to attain its prescribed value 1 at the upper plate.An increase in the permeability of porous medium the temperature in the lower half increases and dicreases in the vicinity of the upper plate(fig.7) Θ enhances with increase in the frequency w(fig .9).when the concentration buoyancy force dominates over the thermal force the temperature enhances with N irrespective of the directions of the buoyancy forces(fig.8).

The concentration distribution(C) for different variations is exhibited in figs.(10-12)

The configuration choosen is such that the molecular diffusibility does not directly affect the flow field and hence the role of schmidt number Sc appears only in the variation of the concentration.We notice that the concentration increases with increase in D-1 or w (fig.10)when the concentration buoyancy force dominates over the thermal buoyancy force the concentration enhances or reduces according as the two buoyancy forces are in the same or opposite directions(fig 10).An

Increase in the molecular diffusivity increases the concentration for $Sc \sim 0.6$ while for higher $Sc \sim 1.3$ C reduces in the fluid region(fig 12).

The shear stress(τ),the Nusselt number(Nu) and the sherwood number on the plates are evaluated for different variations in the governing parameters are presented in tables (1-8).It is observed from tables 1 that the stress component τx increases with a dicrease in N and dicreases with a dicrease in w at z=0.At the upper plate the shear stress dicreases with a dicrease in N and w.As the permeability of the porous medium dicreases the stress (τx) increases at the lower plate and dicreases at the upper plate.(when the concentration buoyancy force dominates over thermal buoyancy the shear stress at both the plates enhances when the two forces are in the same direction and it dicreases when they are in the opposite direction.shear stress τx at both the plates dicreases when N is dicreasing and increase when w dicreases.)from table 2 we find that the shear stress dicrease when N and w both are dicreasing.from table 3 we find that the shearstress τy increases at both the plates when the concentration buoyancy force dominates over the thermal buoyancy force dominates over the thermal buoyancy force dominates over in $|\tau_y|$ increases with dicrease in N and w both are dicreases in permeability of the porous medium $|\tau_y|$ increases when they are in the same direction and it dicreases in opposite directions.At the upper plate $|\tau_y|$ increases when they are in the same direction and it dicreases in opposite directions.At the upper plate $|\tau_y|$ increases with N irrespective of the directions of the buoyancy forces.

The rate of heat transfer (Nusselt number) at both plates are presented in tables 5&6. The rate of heat transfer (Nu) at both the plates increases with increase in thermal buoyancy(G). A dicrease in the permeability of the medium enhances Nu at the lower plate and |Nu| at the upper plate. When the concentration buoyancy dominates over the thermal buoyancy force the rate of heat transfer at the lower plate experiences an enhancement when the forces are in the same direction and reduces when they act in opposite directions. We find a reversed effect at the upper plate. At the lower plate Nu dicreases with N.

The rate of mass flux (sherwood number)at the plates are exibited in tables 7&8 for different variations in the governing parameters.we find that the rate of mass flux at lower plate dicrease with N and w and inicrease at the upper plate.when the concentration buoyancy force dominates over thermal force the sherwood number enhances or reduces at both the platesaccording as the two forces are either in the same or opposite directions.at the lower plate (sh) dicreases with a dicrease in N and increases with a dicrease in N and w.



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CONCLUSION

The steady flow of micro polar fluid in a pipe with porous walls will make the velocity profile lead to high significant consequence. So concluding with the resultant graphs of porous medium.



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